Test Four

Semester Two 2018
UNIT 2 METHODS

Calculator Assumed 40 minutes   /41 marks

Scientific Calculator, ClassPad, Formula Sheet and One page one side of A4 notes is permitted

Name: *Solutions.*

Place a tick in the box next to your Mathematics teachers name:

Mr Strain   ☐

Ms Sindel   ☐

Ms Rimando   ☐

Mr Gannon   ☐

Mr Young   ☐

Mrs Flynn   ☐

Ms Ensly   ☐

Question 1   **(1, 1 = 2 marks)**

A committee of two is to be chosen from a class of 20 students, 12 boys and 8 girls

    i)   How many different committees could be chosen?

$$^{20}C_2 = 190 \checkmark$$

    ii)   How many committees if both students are girls?

$$\binom{8}{2} = 28 \checkmark$$

Question 2   **(1, 1, 1 = 3 marks)**

Six students, three boys and three girls, are to be seated in a row of six seats.

Find the number of ways in which they can be seated:

    i)   if there are no restrictions

$$6! = 720 \checkmark$$

    ii)   if the boys must sit together and the girls must sit together

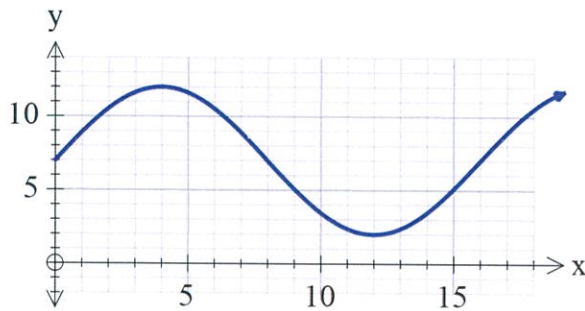$$(3 \times 2 \times 1 \times 3 \times 2 \times 1) \times 2 = 72 \checkmark$$

    iii)   if the boys must sit together.

$$(3 \times 2 \times 1 \times 3 \times 2 \times 1) \times 4 \checkmark$$
$$36 \times 4 = 144$$

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Question 3   **(1, 1, 3 = 5 marks)**

Consider the graph below

i)   State the amplitude of the graph.

5 ✓

ii)   State the period of the graph

16 ✓

iii) Write an equation that will result in the graph.

$y = 5$ ✓ $\sin \dfrac{\pi x}{8}$ ✓ $+ 7$ ✓ or $5\cos\left(\dfrac{\pi x}{8} - \dfrac{\pi}{2}\right) + 7$ ✓

Question 4   **(2, 2 = 4 marks)**

a) Simplify into index form.

$3^{3x+10} \div 15^{4x-6} = \dfrac{3^{3x+10}}{5^{4x-6} \cdot 3^{4x-6}}$

$= \dfrac{3^{3x+10-4x+6}}{5^{4x-6}} = \dfrac{3^{16-x}}{5^{4x-6}}$ ✓✓

b) Solve

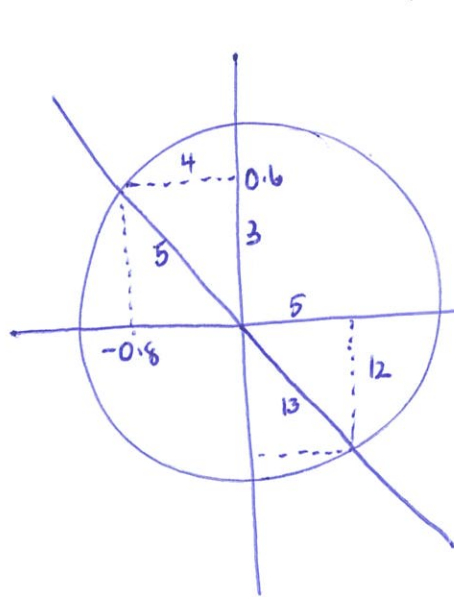$\sqrt{2}\cos 2x - 1 = 0$ for $-2\pi \leq x \leq 0$.

$\text{Solve}\left(\sqrt{2}\cos(2x) - 1 = 0 \mid -2\pi \leq x \leq 0\right)$

$-\dfrac{15\pi}{8},\ -\dfrac{9\pi}{8},\ -\dfrac{7\pi}{8},\ -\dfrac{\pi}{8}$ ✓✓

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If $\sin(x) = \frac{3}{5}$ and $\cos(y) = \frac{5}{13}$, where x is in the second quadrant and y in the fourth, find the exact value of $\sin(x - y)$.

$\sin x = \frac{3}{5}$

$\cos x = -\frac{4}{5}$ ✓

$\cos y = \frac{5}{13}$

$\sin y = -\frac{12}{13}$ ✓

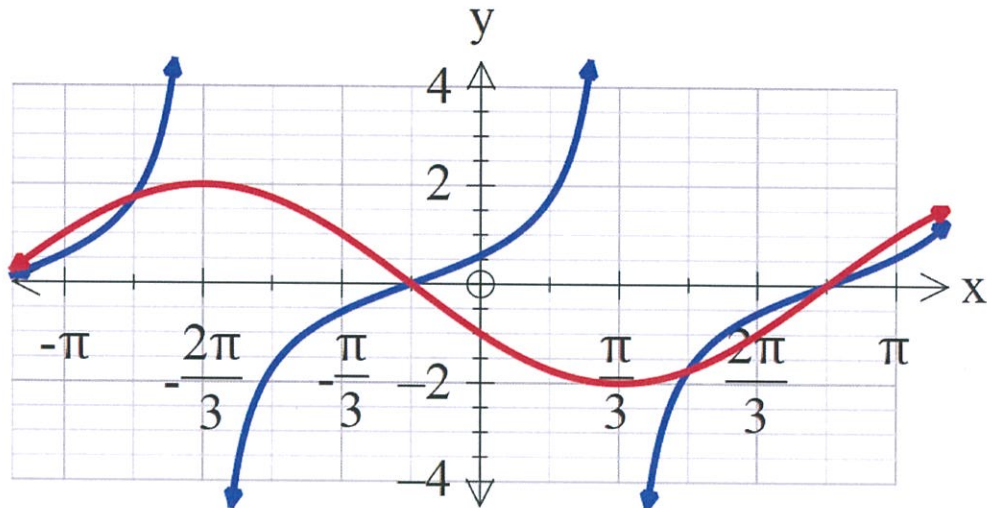$$\sin(x-y) = \sin x \cos y - \sin y \cos x$$

$$= \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) - \left(-\frac{12}{13}\right)\left(-\frac{4}{5}\right) ✓$$

$$= \frac{15}{65} - \frac{48}{65}$$

$$= -\frac{33}{65} \quad ✓$$

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Question 6   (4, 2 = 6 marks)

Shown below are the graphs of

$f(x) = \tan(ax + b)$ and $h(x) = e\cos(x + f)$ where x is in radians.

i)   Determine the values of the constants a, b, e and f.
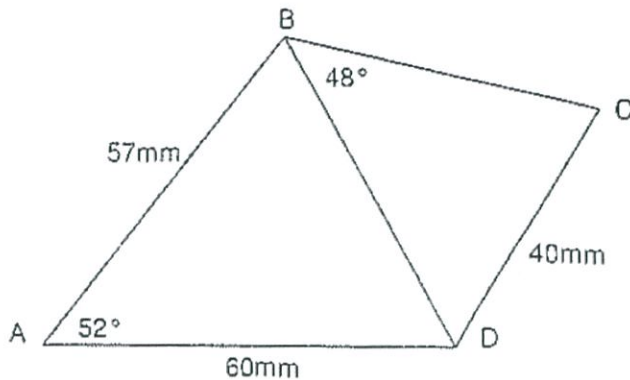
$a = 1$ ✓

$b = \frac{\pi}{6}$ ✓

$e = -2$ ✓

$f = -\frac{\pi}{3}$ ✓

ii)   Use the graph to solve $f(x) = h(x), -\pi \leq x \leq \pi$.

$-\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$ ✓✓

Question 7   (2, 2, 1, 3 = 8 marks)

Consider quadrilateral ABCD, with diagonal BD dividing the quadrilateral into two acute-angled triangles.

a) Calculate the length of the diagonal BD.

$$c^2 = a^2 + b^2 - 2ab \cos C$$
$$= 57^2 + 60^2 - 2(57)(60) \cos 52° \checkmark$$
$$c = 51.36 \text{ mm} \checkmark$$

b) Give the possible sizes of angle C.

$$\frac{\sin C}{51.36} = \frac{\sin 48}{40}$$
$$\sin C = 0.9542 \checkmark$$
$$c = 72.59° \checkmark \text{ or } 107.41° \checkmark$$

c) Why must one of the angles be discarded?

107.41° must be discarded as both triangles are acute. $\checkmark$

d) Calculate the area of quadrilateral ABCD.

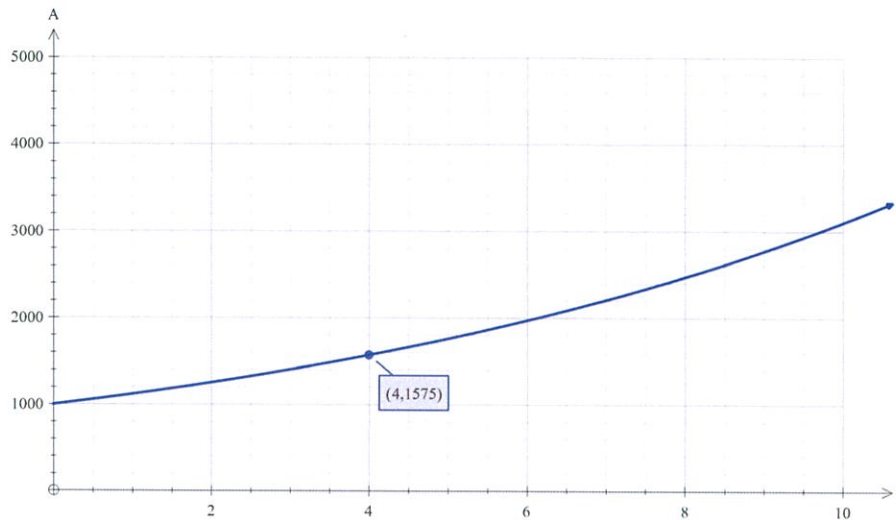$$\text{Area} = \tfrac{1}{2}ab \sin C + \tfrac{1}{2} ab \sin C \qquad \checkmark\checkmark$$
$$= \tfrac{1}{2}(57)(60) \sin 52 + \tfrac{1}{2}(51.36)(40) \sin 59.41$$
$$= 1347.5 + 884.25$$
$$= 2231.7 \text{ mm}^2 \checkmark$$

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Question 8 (1, 1, 2 = 4 **marks**)

The graph below shows the number of assaults in a particular suburb since 1980.

a) Find an exponential model for the number of assaults each year where t is the time since 1980.

$$A = 1000 r^t$$
$$1575 = 1000 r^4$$
$$\therefore r = 1.12 \text{ or } -1.12$$
$$\therefore A = 1000 (1.12)^t \checkmark$$

b) Assuming there was no intervention set up for this suburb, how many assaults would be predicted for 2020?

$$A = 1000 (1.12)^{40}$$
$$\approx 93\,051 \text{ assaults} \checkmark$$

c) When will the assault level have increased 100-fold?

From 1000 to 100 000
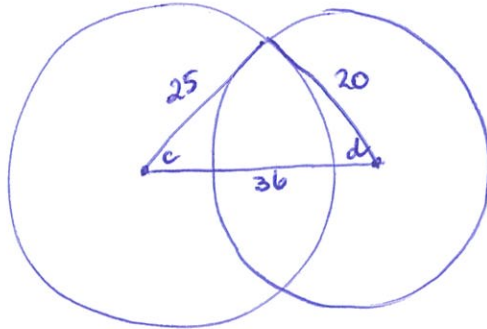
$$100\,000 = 1000 (1.12)^t \checkmark$$
$$t = 40.63$$
$$\approx 41 \text{ years}$$

$$\therefore \text{In } 2021. \checkmark$$

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Question 9   **(5 marks)**

Two circles with radii 25cm and 20cm have their centres 30 cm apart. Determine the size of the common area to both circles correct to nearest square centimetre.

Big Circle

$$\cos C = \frac{c^2 - a^2 - b^2}{-2ab}$$

$$= \frac{20^2 - 25^2 - 30^2}{-2(25)(30)} \checkmark$$

$$\cos C = 0.75$$

$$C = 0.7227 \text{ RAD} \checkmark$$

$$2c = 1.4455 \text{ RAD} \checkmark$$

$$A_1 = \frac{1}{2}(25) \times (1.4455 - \sin 1.4455)$$

$$= 141.67 \text{ cm}^2 \checkmark$$

Small Circle

$$\cos d = \frac{25^2 - 20^2 - 30^2}{-2(20)(30)}$$

$$= 0.5625$$

$$d = 0.9734 \text{ RAD}$$

$$2d = 1.9468 \text{ RAD}$$

$$A_2 = \frac{1}{2}(20)^2 \times (1.9468 - \sin 1.9468) \checkmark$$

$$= 203.33 \text{ cm}^2$$

$$\therefore \text{ Total Area is } 345 \text{ cm}^2 \checkmark$$

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